QcBits:
constant-time small-key code-based cryptography

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Coding theory

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Linear codes

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- compute e (or c) given $c + e$, where e is of weight $\leq t$

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Decoding

- compute e (or c) given $c + e$, where e is of weight $\leq t$
- compute e given the syndrome $He = H(c + e)$

Code-based encryption

- McEliece versus Niederreiter

	plaintext	ciphertext
McEliece	c	$c + e$
Niederreiter	e	$H^* e$

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- General shape

McEliece/Niederreiter + **some code**

Binary-Goppa and QC-MDPC McEliece/Niederreiter

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Confidence	unbroken since 1978	unbroken since 2013

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Key size	$\approx$ 100 kilobytes	$\approx$ 1 kilobyte

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2013 QC-MDPC McEliece (ISIT)

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The problem is timing attacks.

- PQCrypto 2014: constant-time operations assuming no caches
- QcBits: constant-time for a wide-variety of 32/64-bit platforms

Cache-timing attacks

C
C
C
C
C
C

Cryptographic software overwrites some cache lines.

Cache-timing attacks

C
A
A
A
C
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Adversarial software overwrites some cache lines.

Cache-timing attacks

C
A
A
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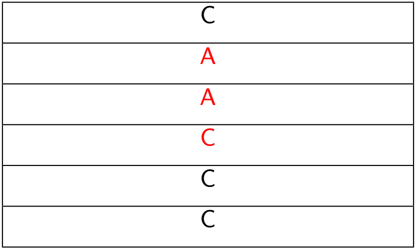
Cryptographic software accesses a cache line.

Cache-timing attacks



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Adversary gains information about the index from timing.

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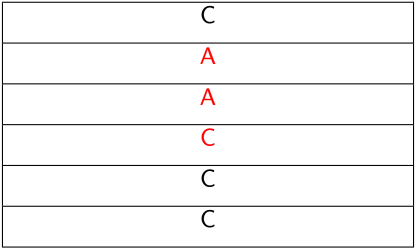

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Solution: don't use secret memory indices.

Performance results

platform	key-pair	encrypt	decrypt	reference	scheme
Haswell	784 192	82 732	1 560 072	**(new) QcBits**	KEM/DEM
	14 234 347	34 123	3 104 624	ACMTECS 2015	McEliece
Cortex-M4	140 372 822	2 244 489	14 679 937	**(new) QcBits**	KEM/DEM
	63 185 108	2 623 432	18 416 012	PQCrypto 2016	KEM/DEM
	148 576 008	7 018 493	42 129 589	PQCrypto 2014	McEliece

Cycle counts for key-pair generation, encryption, and decryption for 80-bit pre-quantum security. Numbers in RED are non-constant-time. Numbers in BLUE are constant-time.

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$$\begin{pmatrix} H^{(0)} & H^{(1)} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \in \mathbb{F}_2^{n \times 2n}$$

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QcBits:

- $[n = 4801, w = 90, t = 84]$ for 80-bit security
- further requires $H^{(i)}$ to have row weight $w/2$
  (same for the Bochum papers)

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- parity$=0$: perhaps no errors. no information.
- parity$=1$: one score for each possible position.

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High-level view

- compute the syndrome
- compute the “probability” that each position is in error
- flip the ones with “higher” probability
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- always work? (probably not)

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- constant-time iterations?

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$$(x - \alpha_i) \mid \mathscr{E}(x) \text{ iff } e_i = 1$$

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- Root finding

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- Bochum strategy: compute u_0, flip v_0, compute u_1, flip u_1, etc.

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$$f, g \in \mathbb{F}_2[x]/(x^n - 1)$$

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$$s = v^{(0)} f + v^{(1)} g \in \mathbb{F}_2[x]/(x^n - 1)$$

Sparse-times-dense polynomial in $\mathbb{F}_2[x]/(x^n - 1)$

Compute $vf \in \mathbb{F}_2[x]/(x^n - 1)$

- v dense, represented as b-bit words (typically $b = 32/64$)
- f sparse, represented as $I_f = \{i \mid f_i = 1\}$

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QcBits computes vf as

$$x^{i_1} v \ + \ x^{i_2} v \ + \ \cdots$$

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- Constant-time rotations?

Barrel Shifter

Rotating by $i = (i_k i_{k-1} \ldots i_0)_2$ bits:

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- conditionally rotate by 2^k bits.
- conditionally rotate by 2^{k-1} bits, and so on.
- Example for $i = 010011_2$ and polynomial

$$(x^8 + x^{10} + x^{12} + x^{14}) + (x^{16} + x^{17} + x^{20} + x^{21}) + (x^{24} + x^{25} + x^{26} + x^{27}) + (x^{36} + x^{37} + x^{38} + x^{39})$$

Barrel Shifter

Rotating by $i = (i_k i_{k-1} \dots i_0)_2$ bits:

- conditionally rotate by 2^k bits.
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- Example for $i = 010011_2$ and polynomial

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010011_2	00110011_2	00001111_2	11110000_2	00000000_2	01010101_2
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$$f, g \in \mathbb{Z}[x]/(x^n - 1)$$

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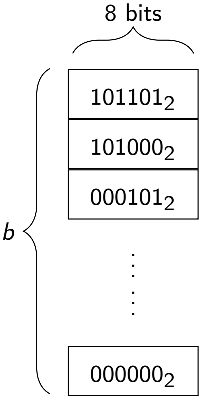
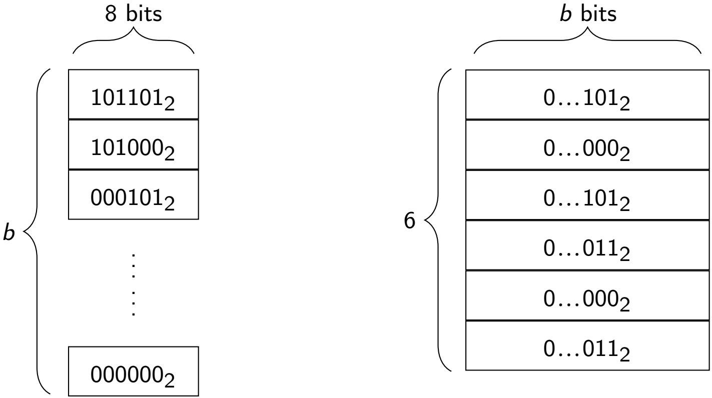
Accumulating $x^i s$

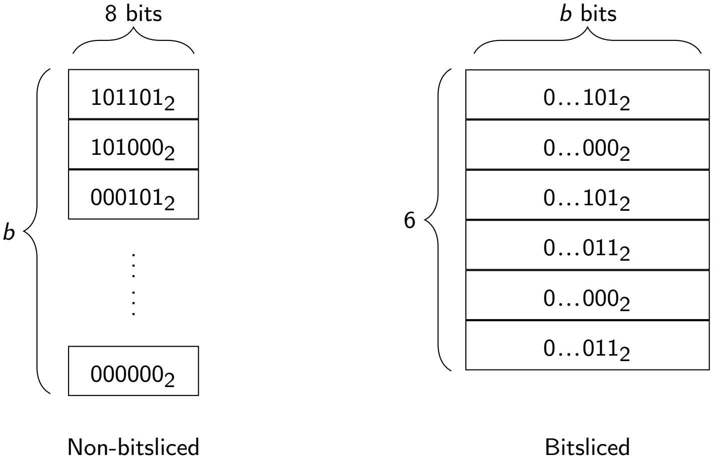
Accumulating x^is



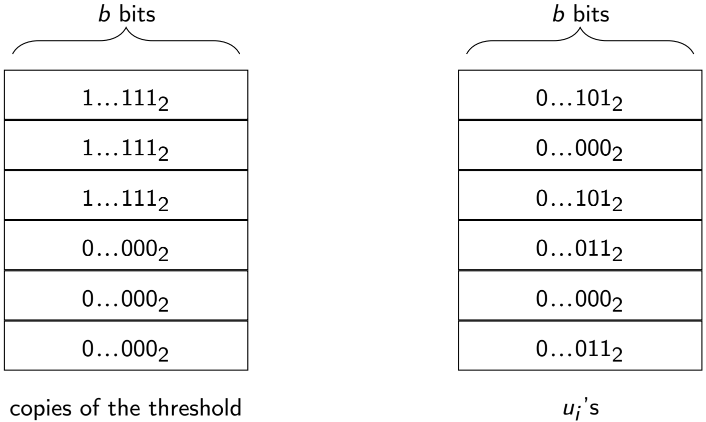
Accumulating $x^i s$



Accumulating x^is



Flipping bits



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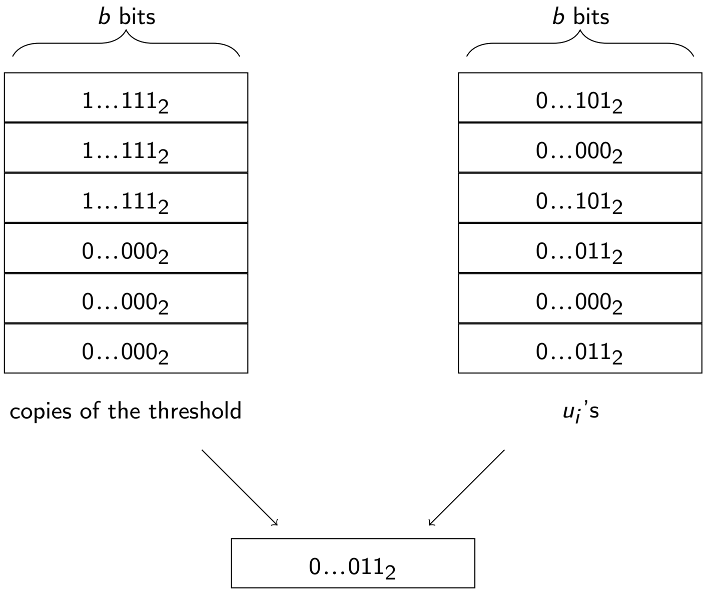

Complexity

Syndrome computation

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Computing the vector u

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$$H^* e = \begin{pmatrix} \mathbf{I} & & H^{(1)} \end{pmatrix} e = s$$

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$$s = e^{(0)} + he^{(1)} \in \mathbb{F}_2[x]/(x^n - 1)$$

The future of QC-MDPC McEliece/Niederreiter

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More research is required to build up confidence.

www.win.tue.nl/~tchou/qcbits/